Precisazioni sul lavoro

"A RELATIVISTIC POINT OF VIEW ON SLIDING FRICTION" presentato nel workshop http://www.wormal.le.isac.cnr.it/program.php

in ricordo di Vincenzo.

Come ho illustrato nel mio intervento al workshop, il lavoro ivi presentato è l'ultimo problema del quale ci siamo occupati insieme Vincenzo, Lorenzo ed io. Vincenzo, a margine della sua attività istituzionale, ma non marginalmente, coltivava da sempre una intensa passione per ogni tipo di problemi e, in particolare, per problemi di matematica, di logica e di fisica. I problemi che trattavamo non erano quasi mai presi da raccolte del tipo di quelle di Martin Gardner o similari ma erano originali in quanto nascevano da personali riflessioni le quali, a loro volta, erano anche stimolate dalle nostre letture di articoli e saggi scientifici. Le discussioni avvenivano generalmente nella stanza di Vincenzo, ma potevano continuare o iniziare anche a due o a tre nella mia stanza o in quella di Lorenzo. Nei nostri lunghi anni di sodalizio abbiamo trattato un discreto numero di problemi. Oltre a discuterne insieme ognuno di noi tentava soluzioni personali e, abbastanza frequentemente, ci trovavamo a seguire approcci diversi. Confrontando i procedimenti risolutivi a volte riconoscevamo che una delle soluzioni era la più elegante. Ognuno prendeva i suoi appunti e le soluzioni più interessanti ce le scambiavamo. Vincenzo rielaborava accuratamente i suoi appunti alcuni dei quali ci forniva in copia dattiloscritta o nella sua chiara grafia, io per lo più i miei li conservavo manoscritti mentre Lorenzo scriveva su dei fogliacci volanti che poi quasi regolarmente perdeva, ciò che lo costringeva a ricostruire ogni volta il ragionamento, esercizio che, secondo me, lo gratificava molto e nel quale era abilissimo. Per fortuna che c'era Vincenzo a conservare le cose importanti! Abbiamo affrontato problemi su molti argomenti diversi, dai facili ma intriganti ai difficili e difficilissimi. Tra questi ultimi, ad esempio, c'erano i famosi Problemi del Millennio che, naturalmente, non abbiamo risolto altrimenti avremmo già intascato i milioni di dollari in palio…Un giorno del 2009 proposi a Lorenzo, che si trovava nella mia stanza, un problema che mi era venuto in mente ad una rilettura del capitolo sull'attrito delle Feynman Lectures. Mi ero chiesto se fosse possibile

calcolare in modo semplice, anche se approssimato, la temperatura che raggiungevano due solidi che scivolavano l'uno sull'altro. Il calcolo si dimostrò più arduo del previsto e Lorenzo decise di coinvolgere Vincenzo. Ne nacque un lungo e acceso dibattito sui vari aspetti del problema, ma non riuscendo a trovare una soluzione convincente decidemmo di documentarci meglio.
Così scoprimmo che, ovviamente, molti altri si erano occupati del problema, anzi che c'era un'intera disciplina, la Tribologia e, più recentemente, la Nanotribologia, che si prefiggeva proprio di calcolare la temperatura dei materiali in frizione ma anche altri effetti quali le deformazioni, l'usura e l'incidenza dei lubrificanti nonché spiegare i meccanismi attraverso i quali l'energia meccanica si trasforma in energia termica. Tale disciplina però, per la parte che ci interessava, ci sembrava avere più un taglio ingegneristico che fisico, ricorrendo essa a modelli delle superfici che venivano trattati numericamente e anche a formule empiriche per calcolare i suddetti parametri. Noi desideravamo una trattazione più concettuale, basata sui principi fondamentali. Capimmo che per analizzare propriamente anche solo gli aspetti basilari dei fenomeni d'attrito occorreva una trattazione congiunta dinamica e termodinamica. Questo approccio non ci permetteva comunque di calcolare la temperatura e neanche precisamente la distribuzione dell'energia termica fra i due corpi, ma almeno ci consentiva di comprendere in modo semplice la fenomenologia energetica. Non eravamo però completamente soddisfatti delle soluzioni proposte. Anche i lavori di Besson del 2001 e 2003, che riassumevano i lavori precedenti e si basavano su un modello mesoscopico delle superfici in interazione nell'ambito della meccanica newtoniana, non ci soddisfecero del tutto. Allora Lorenzo ebbe l'idea di applicare la Relatività Ristretta al problema perché sembrava che con essa si potesse evitare il ricorso ad un modello delle superfici. L'idea appassionò soprattutto Vincenzo che subito si lanciò ad eseguire quei complicati calcoli che egli sapeva fare con proverbiale destrezza. A questo punto il lavoro proseguì ad opera di loro due anche se, occasionalmente, anch'io partecipavo alle sempre colorite e stimolanti discussioni. L'ultima revisione del lavoro di Lorenzo e Vincenzo è datata 9/12/2009. Vincenzo stava sempre più male e dovette sospendere questo lavoro per potersi curare e concentrare sui suoi impegni istituzionali. Poi Lorenzo è andato in pensione e Vincenzo

non molto tempo dopo è morto. L'idea di Alessandra Lanotte di organizzare un workshop in ricordo di Vincenzo ha rivitalizzato l'interesse di Lorenzo e mio su questo lavoro che abbiamo deciso di riprendere ed elaborare e magari pubblicare, come avrebbe desiderato Vincenzo. La versione qui presentata non è ancora quella definitiva, ma abbiamo voluto lo stesso proporla quale contributo alla memoria del nostro caro amico Vincenzo.

Angelo Ricotta
22/12/2012

P.S. Quando scrivevo le precisazioni di cui sopra io e Lorenzo stavamo elaborando la sesta versione del lavoro che veniva acquisendo una fisionomia abbastanza diversa dall'originale con delle idee innovative. Poi sono accaduti degli eventi che ci hanno impedito di continuare. Per non sprecare tutti questi sforzi fatti mi sono deciso comunque a pubblicare in rete quest'ultima versione del lavoro, anche se ancora incompleta, nella speranza che possa servire d'ispirazione a qualcuno che voglia continuare ad occuparsene.

Angelo Ricotta
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A RELATIVISTIC POINT OF VIEW ON SLIDING FRICTION

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ABSTRACT

After having discussed the physical meaning of the 4-force and stated the relativistic version of the Third Law of dynamics in Minkowski spacetime, two examples of solid bodies sliding over each other are worked out evidencing the role of the time component of the involved 4-forces in describing the energetics of the friction. Even though the velocities of the two bodies are far less than the speed of light, the relativistic treatment seems to offer a natural context where to derive, in a unified approach, at the macroscopic level, the basic properties of both the mechanical and thermal effects of friction with no need for particular models.

1. Introduction

In newtonian mechanics to properly explain the heating of bodies subjected to frictional forces we need to adopt a coupled treatment mechanical and thermodynamic.

It has been pointed out by several authors and particularly by Besson (2001, 2003) that to deal satisfactorily with the energy balance in the phenomena of friction it is at least necessary a mesoscopic model of the interacting surfaces. Here we propose an alternative approach based on special relativity which is model-independent and also didactically valuable. A treatment of kinetic i.e. sliding friction, in the context of relativistic mechanics spontaneously leads to introduce, in the representation of contact interaction between two bodies, two terms necessary to set up a correct energy balance. These terms are the instantaneous variation of the total and rest energy of the involved bodies. While a vector representation of frictional forces in the 3-dimensional space is insufficient to give a full account of both mechanical and thermal effects, within the formalism of special relativity the 4-force contains all the necessary physical information allowing to determine both. A key quantity is the scalar product of **K,** the 4-force acting on a body, and **V** the 4-velocity of it. This quantity has the dimensions of a power and allows us to obtain a local and instantaneous energy balance equivalent to the First Law of Thermodynamics. A purely mechanical 4-force i.e. one that is free from thermal effects, must be orthogonal to 4-velocity. Instead a 4-force having a non zero component along the direction of 4-velocity necessarily has thermal effects or, more generally, produces a variation of the rest energy of the body it acts upon. In fact, such a force is able to modify the mass m of the body that, even if this variation is a very small quantity, is able to justify, for either body experiencing friction, the observed warming.

2. The second principle of dynamics

Hereafter 4-vectors are denoted in bold font, while 3-vectors appear capped by an arrow.

The greek indices run from 0 to 3, with 0 reserved for the time component.

Latin indices running from 1 to 3 will be reserved for the three space-components.

The 4-quantities are denoted with capital letters and the 3-quantities with the small ones.

The Lorentz factor $\gamma(v) \equiv (1 - v^2/c^2)^{-1/2}$ is the ratio between the relative dt and the proper time $d\tau$ interval, $dt = \gamma d\tau$, and then it is also the proportionality factor relating the spatial projection of the 4-force to the 3-force $K^i = \gamma f^i$. The same relation holds between the 4-velocity and the 3-velocity $V^i = \gamma v^i$ and $V^0 = \gamma c$. The quantity v appearing in the Lorentz factor is the absolute value of the 3-velocity $\vec{v}$ with respect to the given observer.

The 4-momentum of a moving body, defined as $\mathbf{P} = m\mathbf{V}$, where **V** is the 4-velocity and m the mass, obeys the relativistic second law of mechanics

(2.1) $$\frac{d\mathbf{P}}{d\tau} = \frac{d(m\mathbf{V})}{d\tau} = \mathbf{K}$$

where **K** is the 4-force.

Hence

(2.2) $$m\frac{d\mathbf{V}}{d\tau} + \mathbf{V}\frac{dm}{d\tau} = \mathbf{K} = \mathbf{K}_{\perp} + \mathbf{K}_{\parallel}$$

having distinguished in the 4-force the normal and the parallel component with respect to the direction of the 4-velocity $\mathbf{V}$. The orthogonal component $\mathbf{K}_{\perp}$ of the 4-force determines the 4-acceleration while the parallel component $\mathbf{K}_{\parallel}$ of the 4-force can change the mass m of the body, namely can display an action analogous to that of a heat source or contributes to modify the rest energy of the body.

By scalar multiplying (2.2) with $\mathbf{V}$, recalling that the 4-velocity is a constant-length vector whose squared modulus is $\mathbf{V} \cdot \mathbf{V} = -c^2$ (c = light speed in vacuum) and that the rest energy is $E_0 \equiv mc^2$, it follows

(2.3) $$\frac{dE_0}{d\tau} = -\mathbf{K} \cdot \mathbf{V} = -K^{\alpha}V_{\alpha} = -\left(K^0 V_0 + K^j V_j\right) = K^0 \gamma c - \left(\gamma f^j\right)\left(\gamma v_j\right) = \gamma^2 \left(\frac{K^0 c}{\gamma} - f^j v_j\right)$$

or

(2.4) $$\frac{dE_0}{dt} = \frac{dE_0}{\gamma d\tau} = K^0 c - \gamma f^i v_i$$

This is the law governing the variation of the body’s rest energy.

This energy flow need not represent a heat but may consist of an input of elastic or of other forms of energy.

From (2.1) for the spatial components we have

(2.5) $$\frac{d}{dt}(\gamma m v^i) = f^i$$

and for the time component

(2.6) $$\frac{d}{dt}(\gamma m c) = \frac{K^0}{\gamma}$$

The (2.6) can be rewritten

(2.7) $$\frac{dE}{dt} = \frac{K^0 c}{\gamma} = \frac{dE_0}{\gamma dt} + f^i v_i$$

where $E \equiv \gamma m c^2$ is the total energy.

This is the Law of Energy Conservation from the standpoint of a given inertial observer.

3. The physical meaning of the 4-force

In the absence of ordinary forces, that is in a reference frame where $K^1 = K^2 = K^3 = 0$, but the time component is non zero, $K^0 \neq 0$, a body at rest with respect to a given inertial observer R, according to (2.3, 2.4), varies its rest energy E_0. In such a case the component K^0 of the 4-force represents the equivalent of a heat source or sink. As apparent from (2.3, 2.4) also the three spatial components of the 4–force may concur to change the rest energy of a body. Because the 4-force transforms as any 4-vector, for another inertial observer R' e.g. moving along the *x*-axis with a constant velocity u with respect to R and then the body moves with constant velocity $v = -u$, the corresponding Lorentz transformation produces a space component $K^{x}{}' \neq 0$ and the time component also will assume a different value for the new observer

(3.1) $$K^0{}' = \gamma_u K^0 \qquad\qquad K^x{}' = -\gamma_u \frac{u}{c} K^0$$

The equations (3.1) make clear that if for an observer a body at rest is subjected to heating and no forces act on it, from the viewpoint of another inertial observer R', a non zero 3-force, given by the second one of equations (3.1), appears to act on the body and, actually, it is needed to explain the variation of momentum due to the change in the body's mass produced by the action of the time component of the 4-force.

Since the mechanical power $\vec{f} \cdot \vec{v}$ developed by the 3-force is also a relative quantity, both terms in the energy balance (2.3, 2.4) depend on the observer. Thus what in a reference frame seems to be a mere heating or cooling process, in another reference appears as accompanied by the deployment of mechanical power due to a 3-force acting on the body.

Let us make an example to illustrate more clearly this point. Consider a hot ball in empty space in its rest frame, with no ordinary forces acting on it. The ball keeps cooling by irradiating electromagnetic waves. For another inertial observer moving at a relative velocity u the ball has a non zero, constant velocity $-u$, but, since the ball is continually losing thermal energy, its mass m decreases, and so does its momentum $-m\gamma_u u$, since the product $\gamma_u u$ is constant. This decrease of the linear momentum of the body is produced by the force defined in the second one of equations (3.1) that opposes to the ball's motion but without reducing its velocity. Now where is, physically, this force? The answer is in the asymmetry, for the new observer, of the radiation field emitted by the ball. This asymmetry between front-radiated and rear-radiated waves implies a double Doppler shift in frequency, a blue shift for the photons emitted ahead, and a red shift for those emitted astern. Since the momentum of a photon is directly proportional to its frequency, the combined action of front and rear emissions reduces to a net push opposing the motion of the ball.

To make our point better, let us discuss few other examples. Consider first a constant 3-force $\vec{f}$ of purely mechanical origin that for a given observer pushes or pulls a body varying its kinetic energy, but not its mass ($\frac{dE_0}{d\tau}=0, \vec{f}\neq\vec{0}$). The time component of the corresponding 4-force, because of (2.3, 2.4), cannot be zero, $K^0\neq 0$, and must compensate the power deployed by the force $\vec{f}$, namely, the time component in this case has no thermal effects. Instead, in a reference frame where a body at rest is simply heated ($\frac{dE_0}{d\tau}>0,\ \vec{f}=\vec{0}$) there must be again a non zero K^0, this time accounting just for its warming which implies that the action of $K^0\neq 0$ is now of purely thermal nature. Finally, for a body whose mass is allowed to change in time and which, at the same time, is subjected to the action of a 3-force ($\frac{dE_0}{d\tau}>0, \vec{f}\neq\vec{0}$), K^0 can be both different from zero or zero.

If $K^0\neq 0$ in virtue of (2.7) the total energy varies because of the variations of both kinetic and rest energy. Instead if $K^0=0$, in virtue of (2.7), the total energy is conserved, so that, in view of (2.3, 2.4), all the work done by the force $\vec{f}$ ought to be exactly balanced by the rest energy variation of the body. Thus, in this case we see both mechanical power deployed and heat transfer accomplished, but, despite the presence of both mechanical and thermal effects, we have a time component of the 4-force that is zero.

Actually, Eq. (2.7) tells us just one simple thing about the physical meaning of the time component K^0 of the 4-force: in any situation it must give the total variation of energy of the body, in its broadest sense.

Let us now examine in some detail the motion of a body of mass $m = \text{constant}$ pushed along a fixed direction by a constant 3-force $\vec{f}\left(f\ ,0,0\right)$. In newtonian mechanics the body will accelerate in that direction with constant acceleration, with no other associated effects. What is the equivalent in relativistic mechanics? The corresponding 4-force is $\mathbf{K}\left(K^0,K^1,0,0\right) = \mathbf{K}\left(K^0,\gamma f\ ,0,0\right)$ where the time component K^0 is to be determined.

Putting $m = \text{constant}$ into the equations of motion (2.5) and (2.6), in view of (2.4), we get

$$(3.2) \qquad m\frac{d\left(\gamma v\right)}{dt} = f \qquad m\frac{d\gamma}{dt} = \frac{K^0}{\gamma c} = \frac{\vec{f}\cdot\vec{v}}{c^2}$$

where $\vec{v}\left(v\ ,0,0\right)$ is the unknown velocity of the body.

Now, substituting in (3.2) the second eq. into the first we obtain $m\gamma\frac{dv}{dt} + \frac{f v^2}{c^2} = f$ a nonlinear differential equation in the velocity $v(t)$. The first of (3.2), being m and f constants, can be soon integrated $\gamma v(t) = \gamma_0 v_0 + \frac{f}{m}t$ or $v(t) = \dfrac{\gamma_0 v_0 + f\,t\,/\,m}{\sqrt{1+\dfrac{\left(\gamma_0 v_0 + f\,t\,/\,m\right)^2}{c^2}}}$

whence $\gamma^2(t) = 1 + \dfrac{\left(\gamma_0 v_0 + f\,t\,/\,m\right)^2}{c^2}$.

From the second equation of (3.2) substituting the above γ one has

$$K^0 = mc\gamma\frac{d\gamma}{dt} = \frac{1}{2}mc\frac{d}{dt}\gamma^2 = \frac{f}{c}\left(\gamma_0 v_0 + \frac{f\,t}{m}\right)$$

From this relation it is seen that the original assumption of a constant mass has a well defined aftermath on the time behavior of the time component of the 4-force: the latter cannot be a constant but has to be a linear function of time.

4. The action-reaction principle in relativistic mechanics

In relativistic mechanics the Principle of Action and Reaction (PAR, in short) cannot be the same as in Newtonian mechanics since in this context the concept of immediate action at distance must be abandoned and the simultaneity among events loses its absolute character. The PAR maintains however its validity in collision phenomena and, more generally, in all contact interactions.

So we need a statement of the PAR, valid for contact interactions, where the full 4-forces appear, a statement that must hold true even in those cases where the speeds of the interacting bodies are different from each other. We will now show that in contact interactions, while the 3-forces still obey the PAR in the ordinary sense, the two associated 4-forces of action and reaction are generally different, though collinear. To demonstrate this let us consider only two bodies that collide each other. Then, if we can ignore any other form of energy involved in the collision, e.g the production of particles, radiation emission, and so on, the total linear momentum is the sum of the two single momenta $\mathbf{P}_1$ and $\mathbf{P}_2$ of the two bodies and it is conserved in time for any inertial observer:

(4.1) $$\frac{d}{dt}\left(\mathbf{P}_1+\mathbf{P}_2\right)=0 \ . \qquad \text{(conservation of total 4-momentum)}$$

The relation between the respective proper times of the two bodies, τ_1 and τ_2, and the local observer's time t is $dt=\gamma_1 d\tau_1=\gamma_2 d\tau_2$, hence (4.1) becomes $\frac{1}{\gamma_1}\frac{d\mathbf{P}_1}{d\tau_1}=-\frac{1}{\gamma_2}\frac{d\mathbf{P}_2}{d\tau_2}$.

But, in virtue of the second law of relativistic dynamics (2.1), the two 4-forces, acting as an action-reaction couple, must obey the following relationship:

(4.2) $$\frac{\mathbf{K}_1}{\gamma_1}=-\frac{\mathbf{K}_2}{\gamma_2}$$

Thus, in general, the two 4-forces are not the opposite of each other, though they must at least be collinear in Minkowski space. The above relation contains as a limiting case the ordinary PAR valid in newtonian mechanics in virtue of the relation $K^i=\gamma f^i$ linking the space components of the 4-force and the components of the associated 3-force that substituted in (4.2) gives $f_1=-f_2$.

Starting from the time component of Equation (4.1)

(4.3) $$\frac{d}{dt}\left(P_1^0+P_2^0\right)=0$$

and exploiting the definitions of energy and momentum one has $P^0\equiv mV^\circ=\gamma mc=\frac{E}{c}$

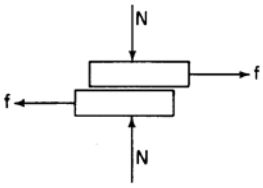
so that it appears that (4.3) is the Conservation Law for the Total Energy of the two-body system according to the given inertial observer $\frac{d}{dt}\left(E_1+E_2\right)=0$.

This null balance reduces to a consequence of the PAR, which, in this way, besides its mechanical implications, assumes also a thermodynamic significance.

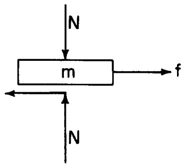
5. Sliding of identical blocks

In all the examples considered here we assume an idealized picture of the real phenomenon of the sliding friction ignoring probable thermal exchanges between the involved bodies and with the environment, deformation and wearing of the sliding surfaces, and so on.

Let us first consider an example taken from Sherwood (1984).



Two identical blocks slide across each other with friction, at a constant speed v and $-v$ with respect to an inertial system, pulled by constant and opposite forces f and $-f$.



Taking into account the upper block we have: the two normal forces N, needed to put in contact the bodies, cancel each other and don't contribute to the warming or the movement, the 4-force corresponding to the 3-force f that pulls the block is $F=\left(F^0,\gamma f,0,0\right)$ and the 4-friction related to the 3-force $-h$ of friction is $H=\left(H^0,-\gamma h,0,0\right)$. The total force acting on the upper block is $K=\left(K^0,K^1,K^2,K^3\right)=\left(H^0+F^0,\gamma\left(f-h\right),0,0\right)$ and the 4-velocity is $V=\left(\gamma c,\gamma v,0,0\right)$.

Note that we consider $f-h>0$ for reasons that will be clear in the following.

From $K=\left(K^0,\gamma\left(f-h\right),0,0\right)$ we have for the internal energy $\dfrac{dE_{0,K}}{dt}=K^0c-K^1v$. Because $v=\mathrm{const}$, from (2.2) it must be $K\parallel V$ i.e. $\dfrac{V^1}{V^0}=\dfrac{K^1}{K^0}$ from which $K^0=\dfrac{K^1}{V^1}V^0$ whence $K^0=\dfrac{\left(f-h\right)\gamma c}{v}$ and eventually

$$\frac{dE_{0,K}}{dt}=K^0c-K^1v=\gamma\left(f-h\right)\frac{c^2-v^2}{v}=\frac{c}{v}\sqrt{c^2-v^2}\left(f-h\right) \tag{5.1}$$

The force F does not contribute directly to the change of the rest energy of the block, it is only the friction H that produces the warming of the block. For this to happen it must be $\dfrac{dE_{0,F}}{dt}=F^0c-F^1v=F^0c-\gamma f v=0$ whence $F^0=\dfrac{\gamma f v}{c}$. From (5.1) it is

$\frac{dE_{0,K}}{dt} = H^0c + F^0c - \gamma f v + \gamma hv$. Substituting $F^0 = \frac{\gamma f v}{c}$ we have $\frac{dE_{0,K}}{dt} = \frac{dE_{0,H}}{dt} = H^0c + \gamma hv$.

Because $K^0 = \frac{(f-h)\gamma c}{v}$ being $K^0 = H^0 + F^0 = \frac{(f-h)\gamma c}{v}$ it will be $H^0 = \frac{(f-h)\gamma c}{v} - \frac{\gamma f v}{c}$ and then $\frac{dE_{0,H}}{dt} = \gamma(f-h)\frac{c^2 - v^2}{v}$. As anticipated above, in the relativistic treatment must be $f - h > 0$ to account for the warming of the block while in the newtonian scheme it is $f = h$.

For the total energy we have, substituting $K^0 = \frac{(f-h)\gamma c}{v}$, $\frac{dE_K}{dt} = \frac{K^0c}{\gamma} = \frac{(f-h)c^2}{v}$.

To verify the above results we can follow a different approach. From (2.5) it is, for the upper block, $\frac{d(\gamma mv)}{dt} = f - h$. Because $v = const \neq 0$ and then $\gamma = const \neq 0$ we have $\frac{dm}{dt} = \frac{(f-h)}{v\gamma}$.

Then $\frac{dE}{dt} = \gamma\frac{dE_0}{dt} = \gamma c^2\frac{dm}{dt}$ whence $\frac{dm}{dt} = \frac{dE_0}{c^2dt}$

and finally

(5.2) $$\frac{dE_0}{dt} = \frac{c^2(f-h)}{v\gamma} = \frac{c^2}{v}\sqrt{1 - \frac{v^2}{c^2}}(f-h) = \frac{c}{v}\sqrt{c^2 - v^2}(f-h)$$

Let us now write the kinetic energy of the upper block

(5.3) $$\frac{dT}{dt} = \frac{dE}{dt} - \frac{dE_0}{dt} = \frac{c^2(f-h)}{v}\left(1 - \frac{1}{\gamma}\right) = \frac{c^2(f-h)}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right).$$

We want to compare the above result (5.3) with its newtonian analogue, e.g. a block in the same conditions as in the relativistic case, whose mass increases in time for some reason. We have $\frac{d(mv)}{dt} = f - h$. Multiplying both members by $\frac{v}{2}$ we obtain $\frac{dT}{dt} = \frac{1}{2}\frac{d(mv^2)}{dt} = \frac{1}{2}(f-h)v$.

Let us now compute the newtonian limit of (5.2) i.e. $v \ll c$. Taking the first order Taylor series of $\sqrt{1 - \frac{v^2}{c^2}} = 1 - \frac{1}{2}\frac{v^2}{c^2}...$ from (5.3) it is $\frac{dT}{dt} = \frac{1}{2}(f-h)v$ which is the same result obtained in the newtonian analogue.

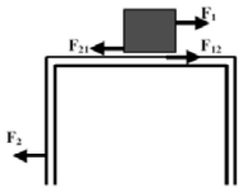
Now we have to note that there would be no sliding friction if there was not present the force F, therefore even if F is not directly responsible for the warming of the block it must be responsible for the variation of the total energy of the block, hence $\frac{dE}{dt} = \frac{F^0c}{\gamma} = fv = \frac{(f-h)c^2}{v}$.

From this last equation we obtain the notable relation $f = \gamma^2 h$ by means of which we can simplify other expressions. The (5.2) becomes $\frac{dE_0}{dt} = \frac{c^2(f-h)}{v\gamma} = \gamma v h$. For $H^0 = \frac{(f-h)\gamma c}{v} - \frac{\gamma f v}{c}$ we obtain the relevant result $H^0 = 0$ and then $K = \left(H^0 + F^0, \gamma(f-h), 0, 0\right) = \left(\frac{\gamma f v}{c}, \frac{\gamma^2 - 1}{\gamma} f, 0, 0\right)$. For the kinetic energy it is $\frac{dT}{dt} = \frac{dE}{dt} - \frac{dE_0}{dt} = f v - \frac{f v}{\gamma} = \left(1 - \frac{1}{\gamma}\right) f v$.

Let us to resume the main results. For the upper block we have $V_u = (\gamma c, \gamma v, 0, 0)$; $H_u = (0, -\gamma h, 0, 0)$; $F_u = \left(\frac{\gamma^3 h v}{c}, \gamma^3 h, 0, 0\right)$ and $f_u = \gamma^2 h$. For the bottom block we have $V_b = (\gamma c, -\gamma v, 0, 0)$; $H_b = (0, \gamma h, 0, 0)$; $F_b = \left(\frac{\gamma^3 h v}{c}, -\gamma^3 h, 0, 0\right)$ and $f_b = -\gamma^2 h$. For both blocks are valid the relations $F^0 = \frac{\gamma f v}{c}$, $H^0 = 0$ and $\frac{dE_0}{d\tau} = fv = \gamma^2 h v$ which is an invariant.

6. A block sliding on a table

Let us consider another example taken from Besson (2001). A block pulled by a constant force f slides with friction at a constant speed v on a table. The table is kept at rest by the force $-f_T$.



The solution for this case can be deduced from the previous one (§ 5.) using the following Lorentz transformations by means of which the previous bottom block becomes the table and then the new inertial reference frame: $A'^0 = \gamma\left(A^0 + \frac{v}{c} A^1\right)$ and $A'^1 = \gamma\left(A^1 + \frac{v}{c} A^0\right)$.

In fact we obtain, transforming the 4-vectors of the bottom block, $V'_b = (c, 0, 0, 0)$ whence $\gamma'_b = 1$; $H'_b = \left(\frac{\gamma^2 h v}{c}, \gamma^2 h, 0, 0\right)$; $F'_b = (0, -\gamma^2 h, 0, 0)$ and $\frac{dE'_0}{d\tau} = \frac{dE_0}{d\tau} = fv = \gamma^2 h v$ being it an invariant.

For the upper block we have, in the reference frame of the table, $V'_u=\left(\gamma'_u c, 2\gamma^2 v, 0, 0\right)$ where

$\gamma'_u=\gamma^2\left(1+\frac{v^2}{c^2}\right)$ from which $v'^1_u=\frac{V'^1_u}{\gamma'_u}=\frac{2v}{1+\frac{v^2}{c^2}}$; $H'_u=\left(-\frac{\gamma^2 hv}{c},-\gamma^2 h,0,0\right)$;

$F'_u=\left(\frac{2\gamma^4 hv}{c},\gamma'_u\gamma^2 h,0,0\right)$ from which $f'_u=\gamma'_u f$ and $\frac{dE'_0}{d\tau}=\frac{dE_0}{d\tau}=fv=\gamma^2 hv$ being it an invariant.

7. Conclusions

In the present paper three results have been obtained.

First, the physical meaning of the 4-force has been clarified both in general and through a detailed example.

Second, starting from the 4-momentum conservation law, we derived the relativistic general version of the Principle of Action and Reaction in the case of pure contact interactions between two bodies, by stating such principle not only for the space components of the two forces of action and reaction, but also for their time components.

Third, to illustrate the case of contact interactions involving kinetic friction, we have discussed in detail two examples. From the discussion of the solutions it appears that the relativistic treatment can shed light in problems where dissipative forces are in action, since the relativistic formalism naturally offers a suitable context for a unified description of both mechanical and thermal effects at the macroscopic level.

Acknowledgments

We have to sadly announce the death of our dear colleague and friend Vincenzo Malvestuto who contributed substantially to this work but could not see its completion.

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