# The true story of the undersampling formulae

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The universally known sampling theorem, credited to Nyquist and Shannon, but the story is more articulated [1] [9], states that to reconstruct correctly the information carried by a bandlimited signal, the sampling frequency $f_S$ must be at least twice the highest frequency $f_H$ of the signal : $f_S \geq 2 f_H$. In practice we use $f_S > 2 f_H$ because there could be ambiguity in reconstructing the component associated to $f_H$ and, depending on the non ideal shape of real-world bandlimited spectra, also folding of the upper part of the spectrum. In this form the theorem is always valid but sometimes it is stated as $f_S > 2B$ where $B = f_H - f_L$ is the bandwidth of the signal. The last formulation implicitly assumes that the lowest frequency $f_L$ of the signal is zero, $f_L = 0$, otherwise it is not generally true, as we will see. If the sampling frequency $f_S$ is lower than $2 f_H$ , i.e. $\frac{f_S}{2} < f_H$, each of the frequencies $f'$ above $\frac{f_S}{2}$ will be aliased, i.e. superimposed or confused, in particular, with a corresponding frequency $f$ in the range $0 \leq f \leq \frac{f_S}{2}$ (page 1 of Fig.1) according to the relation $f' = m f_S \pm f$ or $f = |f' - m f_S|$ with $m = 1,2,3...$ integer (Fig.1).

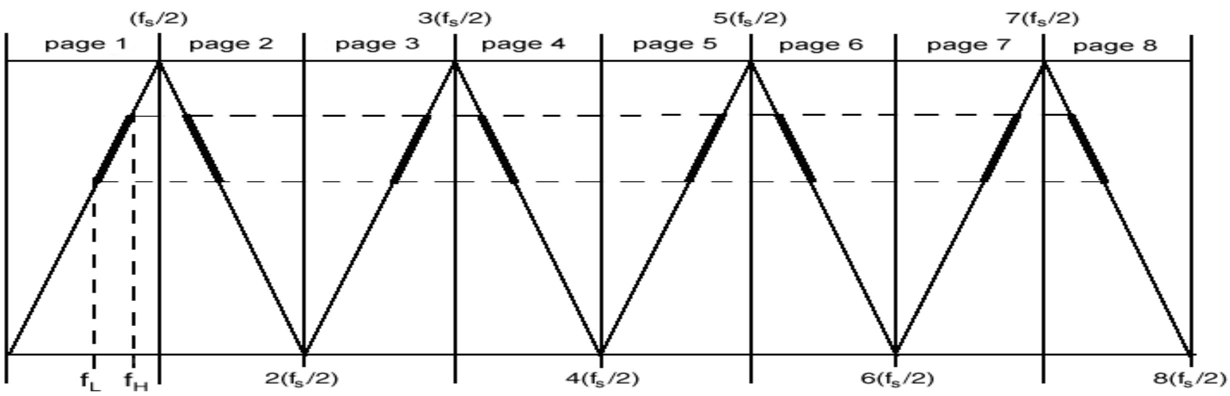



Fig.1 Folding around the Nyquist frequency $f_s/2$ and its multiples [2].

You may think of the diagram of Fig.1 as pages of length $f_S/2$ that fold over each other, in particular over the first, alternately like an “accordion-pleated” [2] strip of paper. Because this phenomenon the spectrum of a signal, when sampled, will be aliased or replicated over all the pages. When the spectrum of the signal is contained entirely in one of the pages, the spectral aliases of the sampled signal will not overlap. If the original spectrum is not on the first page, one of the aliases will be positioned on the first page with the result of having converted down the frequencies of the original spectrum without modification of the bins power. The order of the bins of the original spectrum will be preserved if the original spectrum of the signal is contained in the odd pages and inverted for the even pages. All this occurs because sampling in time domain is a multiplication of the signal by a comb of unitary pulses, which in frequency domain becomes a convolution of the Fourier transformed unitary pulses with the spectrum of the signal. A nice detailed explanation, both mathematical and visual, is given in [4]. The complete spectrum of a sampled bandlimited signal is constituted of replicas of the original spectrum symmetrically disposed around multiples, positive and negative, of the sampling frequency, as illustrated in Fig.2. For example, let the original spectrum (the diagram on the upper side) be composed of two pure tones at 1 and 12 (arbitrary frequency units) and of a continuous spectrum ranging from 6 to 7. Sampling the signal at 5 we obtain the coloured diagram on the bottom, where we can see the symmetrical replicas of the original spectrum centred at multiples of $\pm 5$ and also note that the pure tones, originally positioned at 5 and 12, and then isolated, are now superimposed to the borders of the continuous spectrum. Of course, when we sample a signal with continuous spectrum for a finite duration $t_s$, the spectrum of the sampled signal will be constituted of discrete components (bins), whose frequency resolution is $\Delta f = \frac{1}{t_s}$, which are not visible in Fig.2.

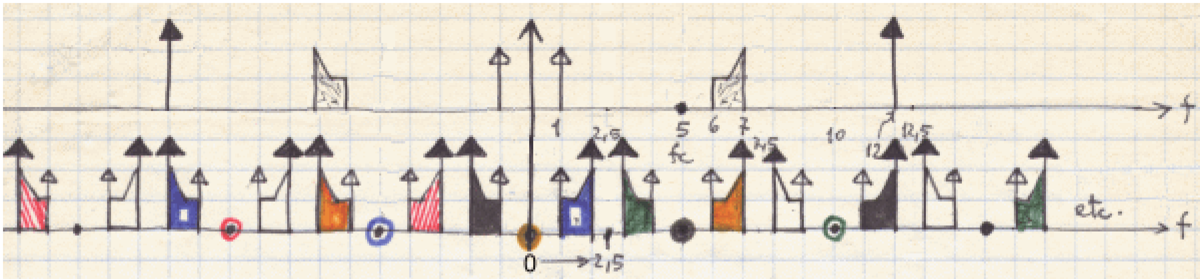



Fig.2 Original drawing (1981) of the replicas of the spectrum of a sampled bandpass signal.

The "accordion-pleated" paper model leads to a straightforward mathematical formulation.
Let $[f_L, f_H]$ be the bandwidth of the signal to be sampled.
The key conditions to avoid the folding of the spectrum on itself, for $n = 0,1,2,3...$ integer, are [3]

$$n\frac{f_S}{2} < f_L \text{ and } f_H < (n+1)\frac{f_S}{2}$$

which means that the original spectrum of the signal must be contained entirely in one of the pages of Fig.1. Actually the spectrum could be segmented in different pages. In that case we have to state the above conditions for each segment and others have to be verified so that the segments will not fold on each other when the signal is sampled.

The connections among the page number, $n$ and $m$ are: $n = page\,number - 1$ and $m = \left\lfloor \frac{n+1}{2} \right\rfloor$ where $\lfloor \arg \rfloor$ is the floor function, i.e. the largest integer not greater than "arg".

Isolating $f_S$ from the above inequalities we have

$$\frac{2f_H}{n+1} < f_S < \frac{2f_L}{n} \qquad (1)$$

and eliminating $f_S$ we obtain $\frac{2f_H}{n+1} < \frac{2f_L}{n}$ from which

$$n < \frac{f_L}{f_H - f_L} \qquad (2)$$

These are the fundamental formulae for undersampling, even if I did not use this term in my original report [3] because, at that time, I was not concerned about any specific terminology for this kind of operation.

For example, let it be $f_L = 1550\,kHz$ and $f_H = 2100\,kHz$. Applying (2) we have $n < 2.8$, i.e. $n = 2,1,0$, and then from (1) we obtain all the allowable sampling frequencies: $n = 2$: $1400\,kHz < f_S < 1550\,kHz$, $n = 1$: $2100\,kHz < f_S < 3100\,kHz$ and, of course, $n = 0$: $4200\,kHz < f_S$. As anticipated, the order of the bins of the aliased spectrum of the bandpass signal is reversed or not depending on the position of the original spectrum of the signal to respect to the chosen $f_S$: if the corresponding $n+1$ is odd the order is preserved, if it is even the order is reversed.
Note that for doing a correct undersampling you cannot use all the sampling frequencies $f_S > 2(f_H - f_L)$, in fact in the above example it would be $f_S > 1100\,kHz$ which is clearly wrong. If you are interested only in the lowest bound $(f_S)_{LB}$ of the sampling frequencies, substituting (2) into (1) the left term furnishes

$$(f_S)_{LB} = \frac{2 f_H}{\left\lfloor \dfrac{f_H}{f_H - f_L} \right\rfloor} \qquad (3)$$

which is the same as that reported in [4]. An expression equivalent to (1), but in the time domain, is given in [5]. We did not use the equal sign in (1) and (2) to avoid a possible folding and ambiguities of the frequencies at the borders of the spectrum, but if the power in the bins outside the open range $(f_L, f_H)$ is zero or negligible we may write $\frac{2f_H}{n+1} \le f_S \le \frac{2f_L}{n}$ and $n \le \left\lfloor \frac{f_L}{f_H - f_L} \right\rfloor$, i.e. we have to consider the shape of the real-world bandpass spectrum and choose $[f_L, f_H]$ so that to avoid ambiguities and minimize the folding of the spectrum on itself. It is possible to give to (3) a different form. Let it be $\frac{f_H}{f_H - f_L} \equiv x$ and call it "band index". Consider $\frac{(f_S)_{LB}}{f_H - f_L} \equiv y = \frac{2x}{\lfloor x \rfloor}$. The range of $\frac{x}{\lfloor x \rfloor}$ is $[1,2)$, furthermore it is $\lim_{x\to\infty} \frac{x}{\lfloor x \rfloor} = 1$ and then $\lim_{x\to\infty} \frac{(f_S)_{LB}}{f_H - f_L} = 2$. The $y$ range is $[2,4)$ and its diagram is shown on Fig.3. Note that for $x = 1,2,3,4,\ldots \Rightarrow y = 2$ not $4$.

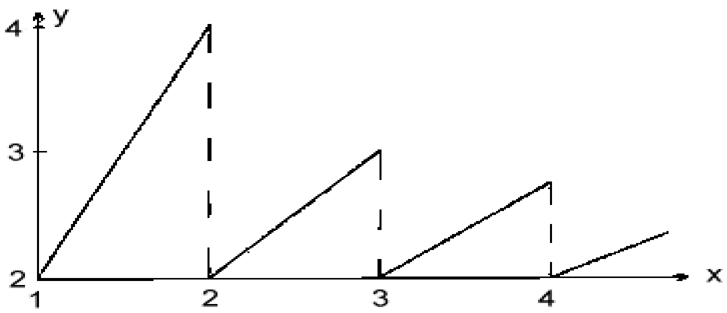



Fig.3 Lowest bound of the sampling frequency normalized to the bandwidth versus the band index.

In a recent article [6] were reported the two formulae $f_{SAMPLE} > 2\Delta f_{SIG}$ and $f_{SAMPLE} = \frac{4 f_{CAR}}{2Z - 1}$ to compute an allowable sampling frequency for undersampling a bandpass signal, being $f_{CAR}$ the carrier, and $\Delta f_{SIG}$ the bandwidth of the signal. The procedure to compute $f_{SAMPLE}$ is: as a first approximation put $f_{SAMPLE} = 2\Delta f_{SIG}$, insert this value in $Z = \left\lfloor \frac{(4 f_{CAR} / f_{SAMPLE}) + 1}{2} \right\rfloor$ then use this rounded-down integer value of $Z$ to calculate the true $f_{SAMPLE}$. I think that this method, as an illustration of the undersampling concepts, is useless and even misleading at least for two reasons: the first because it is not simpler than the more general approach given by the inequalities (1) and (2), the second, and worse, because it gives only a single sampling frequency instead of all the permitted frequencies, and the computed frequency is not even the lowest bound, but only that particular sampling frequency for which the spectral aliases of the sampled signal are centred on the pages of Fig.1. The above formulae express this last property in a foggy way and even as an algorithm they are twisted, compared to the sunny logic of (1) e (2). In fact take $f_{CAR} = \frac{f_L + f_H}{2}$ and $\Delta f_{SIG} = f_H - f_L$, being $f_{CAR} = 1825\,kHz$ and $\Delta f_{SIG} = 550\,kHz$, as in the preceding example, we will have $f_S = 1460\,kHz$, instead the lowest bound for the sampling frequency is $\left(f_S\right)_{LB} = 1400\,kHz$. If the only data at our disposal are $f_{CAR}$ and $\Delta f_{SIG}$ it is easy to switch to the general method taking $f_L = f_{CAR} - \left(\Delta f_{SIG}/2\right)$ and $f_H = f_{CAR} + \left(\Delta f_{SIG}/2\right)$ and carrying on the computation as I suggested. Anyway, the formulae $f_{SAMPLE} > 2\Delta f_{SIG}$ and $f_{SAMPLE} = \frac{4 f_{CAR}}{2Z - 1}$ can be easily deduced from $\frac{2 f_H}{n+1} < f_S < \frac{2 f_L}{n}$. By definition $\Delta f_{SIG} = f_H - f_L$, therefore $f_{SAMPLE} > 2\Delta f_{SIG}$ is always satisfied, because implicitly contained in (1): note that you cannot use every $f_{SAMPLE} > 2\Delta f_{SIG}$ for undersampling, as already shown, because you have to satisfy the other constraint. To deduce $f_{SAMPLE} = \frac{4 f_{CAR}}{2Z - 1}$ from the key conditions [3] consider that the aim of the above formula for $f_{SAMPLE}$ is to centre the spectral aliases of the sampled signal on the pages of Fig.1. It has to be $f_L - \frac{n f_{SAMPLE}}{2} = \frac{(n+1) f_{SAMPLE}}{2} - f_H$ from which $f_{SAMPLE} = \frac{2\left(f_L + f_H\right)}{2n+1} = \frac{4 f_{CAR}}{2n+1}$.

Even in practical applications it is important to be able to calculate **all** the permitted sampling frequencies, because you may have some constraints that force you to choose a particular range of sampling frequencies, so that it is better to rely on the general method for this computation.

My interest in signal processing started in the mid of 1975 when I began doing my thesis in Physics [7] which consisted in the design and in the realization of a SODAR system for use in atmospheric boundary layer studies. For the hardware I basically followed the work done by E.J.Owens [8], adding some original solutions. Anyway I was the first in Italy to design and build a SODAR system that really worked, and even today many people use my scientific and technical ideas and solutions, some of which are described in [3] [7] [10].

During the 1976, and for many years after, the first version of the SODAR, and its upgrades, I designed and built personally, were extensively used in measurement campaigns and there emerged the need of an efficient sampling of the signal and the necessity of a real-time processing of the data. The first need came also from the fact that we had old computers with limited A/D and poor storage units, the second because we needed the wind profiles immediately for certain applications in the air pollution monitoring. The SODAR is capable of producing a cumbersome amount of data even for today standards, especially if you want to store the raw data for advanced future analysis and because you have to digitize the signal continuously for many days, and sometimes for months. So that I had to reduce the rate and the amount of sampled data without losing the information we were interested on. The solution proceeded by successive approximations. My first approach was hardware and I designed, in 1980, an audio heterodyne (p.7) that translated down the spectrum of the echo. It was also tried the decimation of the sampled data, comparing the spectra before and after and observing empirically that, in certain conditions, the result was only a down translation of the frequency bins without modification of the bins power. Then at the beginning of 1981 I ran into [2], p.230, and imagined that the "accordion-pleated" paper model had a useful mathematical formulation in terms of the fundamental formulae (1) and (2) for undersampling shown above. Only much more later I read [4] and [5] and realized that, at least (1) was already known, even if the topic was understated and treated differently (it was never named undersampling but "bandpass sampling theorem") and partially and without proof in the quoted references, instead I think that my proof is simple and smart. In [4] the fundamental formula is stated differently and in the time domain instead of frequency domain, as I did. Furthermore no formula for $n$ is given. In [5] the "bandpass sampling theorem" is listed among the problems left to the reader and the formula shown refers only to the critical sampling frequency (3), but one of the terms may suggest, to an attentive reader, the way to compute $n$. At that time, to my knowledge, people working on SODAR systems did not use the undersampling technique to digitize the signal, and even FFT was not so popular. Hence I think I was the first to introduce the undersampling in this area [3] and in a very simple form well suited for practical use. My fault was not to publicize enough my results with the consequence that a few people have tried to catch the merit for them even people I informed of personally [13].

But, even if my report of 1983 was late, having I achieved the results in 1981 and even before, the papers of the others are all at least of two or more years later and, in a number of cases happened in Italy I know why: at the beginning they did not believe in my results!

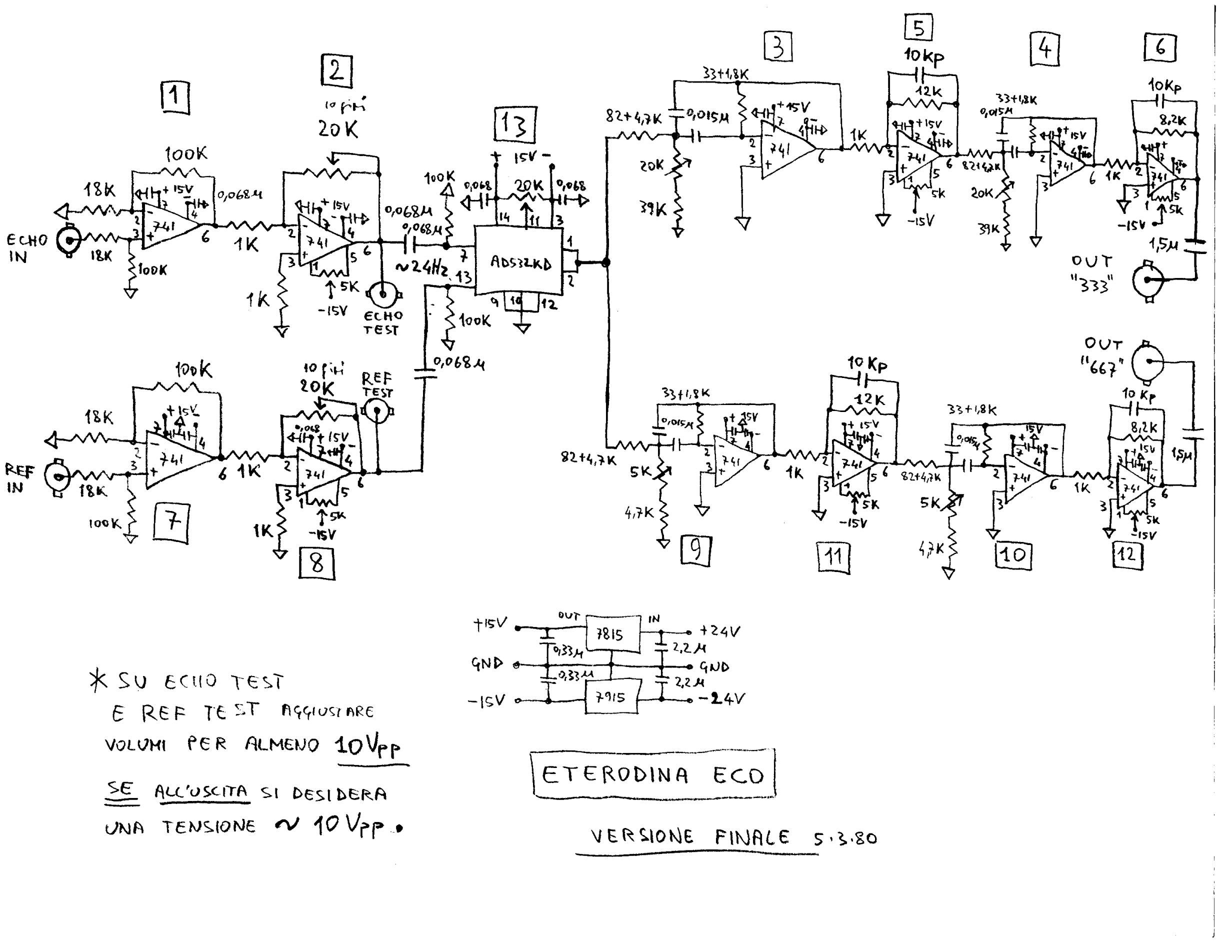
The cold fact is that I have been using the undersampling technique since 1980 and wrote a report [3] where I gave two simple and practical formulae to compute **all** the allowable sampling frequencies for undersampling a given bandpass signal. The report was written in Italian and was known, at least, among the Italian community working on SODAR systems in which a few people and even students utilized my formulae in an unfair way because they did not mention the source. On 10 October (pp.8-9) and 7 December 1991 (pp.10-11), to stop the above misuses, I sent two letters containing my formulae for the undersampling to EDN Signals & Noise Editor but I never received an answer. On 25 March 1994 I attended a Burr-Brown's Applications Seminar in Rome, Italy, where I explained to the two relators my formulae. One of the relators, Mr. Jason Albanus, suggested to me to send my formulae to Mr. Jerry Horn at Burr-Brown Corp. (pp.12-13), Tucson, Arizona, for inclusion in future seminar books. I did this way but my letter was never acknowledged. Then on 11 July 1994, on Electronic Design, appeared an article [13] by George Hill of Burr-Brown Corp., Tucson, Arizona, in which he exposed, at p.77, my formulae for undersampling stating literally: "After a recent applications seminar given by Burr-Brown **in Rome, Italy**, **one of the attendees** suggested an approach for easily calculating appropriate sampling rates for undersampling any specified range of input frequencies. He offered his ideas for inclusion in future seminars, **but didn't authorize us to use his name**. Here is his approach…". Of course I was that attendee and for me was clear that Mr. George Hill and everyone else should have used my name in connection with my formulae! For that on 13 September 1994 I wrote to Mr. George Hill (p.14) inviting him to do so, but again there was no answer.

Many years later (2005) I met in the group comp.dsp Mr. Richard G. Lyons who recognized the plagiarisms (pp.15-16) but even in the 3rd edition (2011) of his famous book [11] he never cites my name or my work [3] in connection with the formulae (1) and (2). In the 1st edition (1997) of his book he quotes only two articles [12] [13] in connection with the formulae (1) and (2) but they were both published many years later of that of my work. I do not know if [12] contains the above formulae but certainly they are in [13] i.e. the article by George Hill! A very unfair behaviour!

REFERENCES

[1] Abdul J. Jerry, "The Shannon Sampling Theorem – Its Various Extensions and Applications: A Tutorial Review", Proc. of IEEE, vol.65, no. 11, November 1977.
[2] Julius S. Bendat, Allan G. Piersol, "Random Data: Analysis and Measurement Procedures", Wiley-Interscience, 1971, (p.230).
[3] Angelo Ricotta, "Some remarks on the sampling and processing of SODAR data", Technical Report, IFA 83/11, IFA-CNR, July 1983, (pp. 4-7, in Italian).
[4] E. Oran Brigham, "The Fast Fourier Transform", Prentice-Hall, Inc., 1974, (p.87).
[5] "Reference Data For Radio Engineers, Fifth Edition", Howard W. Sams & Co., Inc., ITT, 1970, (p.21-14).
[6] Bonnie Baker, "Turning Nyquist upside down by undersampling", EDN 12 May 2005.
[7] Angelo Ricotta, "Development of an acoustic radar and its applications to the planetary boundary layer dynamics studies", Thesis in Physics, University of Rome, Italy, 1976 (in Italian).
[8] E. J. Owens, NOAA MARK VII Acoustic Echo Sounder, NOAA Tech. Mem., Boulder, Colo., 1975.
[9] P. L. Butzer, J. R. Higgins, R. L. Stens, "Sampling theory of signal analysis", Development of Mathematics 1950-2000, Editor J. P. Pier, Birkhäuser, 2000.
[10] A. Ricotta, M. Berico, "Triaxial Sodar", Technical Report LPS 80-6, LPS-CNR, Frascati, 1980.
[11] Richard G. Lyons, "Understanding digital signal processing", 3rd ed., Prentice Hall, 2011.
[12] Steyskal, H. "Digital Beamforming Antennas," *Microwave Journal*, January 1987.
[13] Hill, G. "The Benefits of Undersampling," *Electronic Design*, July 11, 1994.

# Alleged documents



Frascati October 10, 1991

Dear Signals & Noise Editor,

I am referring to the article entitled "Undersampling reduces data acquisition costs for select applications", by Jeff Kirsten and Tarlton Fleming, published on EDN June 21, 1990 and again in July 1991 on Electronic Engineering.
I have been using the undersampling technique since long time ago (A.Ricotta, Some remarks on the sampling and processing of SODAR data, Technical Report, IFA-CNR, July 1983), and I have found useful to use two explicit formulae to quickly compute **all** the permitted intervals of sampling frequencies when $f_{INLOW}$ and $f_{INHIGH}$ of the bandpass input signal are done.
We can write

$$\frac{2}{n+1} f_{INHIGH} < f_{SAMPLING} < \frac{2}{n} f_{INLOW}$$

where

$$n_{INTEGER} \le \left[ \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \right]$$

so that $n_{INTEGER} = 0, 1, 2, \ldots, \max(n_{INTEGER})$ .
At $\max(n_{INTEGER})$ corresponds the interval of the minimal sampling frequencies.
Furthermore, if we choose $n_{INTEGER}$ even, the aliased input signal preserves its spectral order, while for $n_{INTEGER}$ odd, the order is reversed.
For example let $f_{INLOW} = 500 kHz$ and $f_{INHIGH} = 550 kHz$ as in the cited article.
From the above inequalities we obtain

$$n_{INTEGER} \le \frac{500}{50} = 10 \quad \text{(exactly)}$$

hence $n_{INTEGER} = 0, 1, 2, 3, \ldots, 10$: in some applications we must reject the exact maximum $n_{INTEGER} = 10$ to avoid to alias the spectral borders of the input signal with $0$ or $\frac{f_{SAMPLING}}{2}$ .
From each value $n_{INTEGER}$ we can compute an interval of permitted sampling frequencies:

$n_{INTEGER} = 0 \Rightarrow 1100 kHz < f_{SAMPLING}$ ( Nyquist)

$n_{INTEGER} = 1 \Rightarrow 550 kHz < f_{SAMPLING} < 1000 kHz$

and eventually

$\max(n_{INTEGER}) = 10 \Rightarrow 100 kHz < f_{SAMPLING} < 100 kHz$

which is the interval of the minimal permitted frequencies (only one in this case). Because 10 is even the spectral order is preserved.
Generally we choose $\max(n_{INTEGER})$ and then the minimum allowed

$f_{SAMPLING}$, but for particular applications is useful to have the possibility to select different values of $f_{SAMPLING}$ and also of $n_{INTEGER}$.

Yours sincerely,

Address:
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ITALY

Frascati December 7 , 1991.

Dear EDN Signals & Noise Editor,

The reference is my letter of October 10, 1991.
I want to add some information to clarify the expressions

$$\frac{2}{n+1} f_{INHIGH} < f_{SAMPLING} < \frac{2}{n} f_{INLOW} \qquad (1)$$

$$n_{INTEGER} \leq \left\lfloor \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \right\rfloor \qquad (2)$$

in which the symbol $\lfloor --- \rfloor$ means "the whole part of ---" and $n_{INTEGER} = n$ of the expression (1).
This expression for $n_{INTEGER}$, when the band of the input signal is a submultiple of $f_{INLOW}$, as in the example of the article by Kirsten & Tarlton, EDN June 21,1990, implies that

$$\max(n_{INTEGER}) = \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}}$$

This choice produces an alias which is unimportant in the example chosen by Kirsten & Tarlton, as they explicitly stated, but unacceptable in situations in which we are interested in precise measurements even of the spectral borders of a generic input band.
If we want to avoid that kind of alias we have to choose $n_{INTEGER}$ as

$$n_{INTEGER} < \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \qquad (3)$$

For the sake of precision the expression (1) implies (3) while (2) is implied by (1) with ≤ instead of <.
It is straightforward to prove the expressions (1) and (3) starting from the basic concept that our input band must be between $n\frac{f_{SAMPLING}}{2}$ and $(n+1)\frac{f_{SAMPLING}}{2}$.
We can immediately put the above statement in the form

$$\begin{cases} n\frac{f_{SAMPLING}}{2} < f_{INLOW} \\ f_{INHIGH} < (n+1)\frac{f_{SAMPLING}}{2} \end{cases} \qquad (4)$$

hence it is a fortiori

$$f_{INHIGH} < f_{INLOW} + \frac{f_{INLOW}}{n}$$

and eventually we obtain the expression (3), while we have (1) directly from (4).

Bibliography:
a) Reference data for radio engineers, 5th Edition, 1970, Howard W. Sams & Co.-ITT :at p.21-14 there is a version of (1) in the time domain.
b) Bendat J.S. & Piersol A.G., Random data: Analysis and

measurement procedures, 1971, Wiley-Interscience : at p. 230 there is an illustration of the aliasing and an expression equivalent to $f_{ALIASED} = | f_{SIGNAL} - n f_{SAMPLING} |$.

c) Brigham E. O., The fast Fourier Transform, 1974, Prentice-Hall, New Jersey : At p.87 the problem 5-4 contains a formula for the critical sampling frequency obtainable from (4) with the substitutions $n = n' - 1$ and ≤ instead of <.

Yours sincerely,

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Frascati March 28 , 1994

Dear Mr.Jerry Horn
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The 25th of March I attended a Burr-Brown's Applications Seminar, held in Rome, Italy.
We discussed, among other topics, the undersampling technique.
About that, I communicated to the relators two formulae that would be useful to add to the seminar book and elsewhere for practical use. Mr. Jason Albanus suggested to me your name to ask about it.
If we know the lowest $f_L$ and the highest $f_H$ frequency of the input spectrum, we can put the basic principle (e.g. at p.232 of the seminar book) in the form $n\frac{f_S}{2} < f_L$ and $f_H < (n+1)\frac{f_S}{2}$ with $n = 0, 1, 2, \ldots$.
To respect to $f_S$ we have $f_S < \frac{2f_L}{n}$ and $f_S > \frac{2f_H}{n+1}$, hence

$$\frac{2}{n+1} f_H < f_S < \frac{2}{n} f_L \qquad (1^{st} \text{ useful formula})$$

which is the interval of the allowed sampling frequencies.
The above inequalities imply $\frac{f_L}{n} > \frac{f_H}{n+1}$, i.e. $\frac{1}{n} > \frac{f_H}{f_L} - 1$ and finally

$$n < \frac{f_L}{f_H - f_L} \qquad (2^{nd} \text{ useful formula})$$

that gives us a criterion to choose $n$(integer). Besides we can rewrite the formula at p.231 as $f_{SAMPLED} = \left| f_{INPUT} - \left\lfloor \frac{n+1}{2} \right\rfloor f_S \right|$ where the symbol $\left\lfloor \frac{n+1}{2} \right\rfloor$ stands for "floor": the greatest integer not greater than $\frac{n+1}{2}$ .
Furthermore, the order of the spectral bins of the input spectrum is preserved or inverted in the sampled spectrum, depending on the parity of $n$: the order is preserved if $n$ is even, inverted if $n$ is odd.
For example let $f_L = 24.6MHz$ and $f_H = 27MHz$ ,$BW = 2.4MHz$. We have

$$n < \frac{24.6}{2.4} = 10.25$$

Choosing $n = 10$ we obtain

$4.\overline{90}MHz < f_S < 4.92MHz$

We could choose $f_S = 4.91MHz$.
The sampled spectrum would be in the band
$f_{L_{SAMPLED}} = |24.6 - 5*4.91| = 0.05MHz$

$f_{H_{SAMPLED}} = |27 - 5*4.91| = 2.45MHz$

Because $n$ is even, the spectral order of the input spectrum is preserved in the sampled spectrum.
Of course we could also use in principle $n = 0, 1, 2, \ldots,$ until 9.
For $n = 0$ we would obtain $54MHz < f_S$, the Nyquist rate; for $n = 1$ $27MHz < f_S < 49.2MHz$ and so on up to 9: no other sampling frequencies intervals are allowed if we wants to avoid the folding of the sampled spectrum: e.g. $f_S = 4.89MHz > 2BW$ is not allowed.
I would appreciate your comments.

Yours sincerely,

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CONSIGLIO NAZIONALE DELLE RICERCHE
ISTITUTO DI FISICA DELL'ATMOSFERA

Frascati September 13, 1994

Dear Mr. George Hill
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I congratulate you on your clear article: "The benefits of undersampling, Electronic Design, July 11, 1994".
I am the attendee at the Burr-Brown's seminar given in Rome, Italy, on March 25, 1994, to whom you refer in the window "How to determine the proper undersampling rate" at p.77. Of course you, and all other people are authorized to use my name in connection with the equations shown in the window (I thought I implicitly stated it in my previous letter on March 28, 1994, which received no answer).
I have been using the undersampling technique since long, and as far as I know, I was the first, despite their simplicity, to publish the cited equations, in the report: "A. Ricotta, Some remarks on the digitization and processing of Sodar data, Technical Report, IFA-CNR, July 1983". This report was written in Italian, but you can easily recognize at p. 6 the equations under discussion (I will send you this report, by mail, for reference).
I would be grateful to you if you would explicitly acknowledge this my small contribution.
I would appreciate your answer.

Yours sincerely,

(Angelo Ricotta)

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On Sun, 05 Jun 2005 07:09:46 GMT, "erine" <er...@hotmail.com> wrote:
>Read this astonishing story at
>http://spazioscuola.altervista.org/UndersamplingAR/UndersamplingAR.htm
>Here it is an excerpt:
>A STORY ABOUT UNDERSAMPLING
>
>by Angelo Ricotta - Rome, Italy
>
>a.ri...@isac.cnr.it
>
>ITALIAN VERSION
>
>
>In the article "Turning Nyquist upside down by undersampling" by Bonnie
>Baker, EDN 12 May 2005, are reported the two formulae  and  to compute an
>allowable sampling frequency for undersampling a bandpass signal. I was
>surprised by that because I have been using the
  (Angelo's sad story snipped)

Hi Angelo,

  Shame on those Burr-Brown & EDN knuckleheads.

There is *NO* excusing their behavior.
Their plagiarism was a very bad thing to do
in our business.

Many of us here on comp.dsp have had
bad experiences with people plagiarizing
our work.

Weeks ago I saw the Bonnie Baker article and was
a little surprised to see a "bandpass sampling"
sample rate computation scheme (your method) that I
had not seen before.

I experimented with your method (comparing it with
a method that I use to compute Fs) and your
scheme sure seems to work just fine.
So Angelo, "Good work".

I think Bonnie Baker should be made aware of your
story and she should tell the "real story" of the
origin of the method she included in her
article.

Angelo, to try to "make up" for the way you were
treated back in the 1990s, let me know if you'd like
me to help you convince Ms. Baker to tell your story.
(Not that I have any influence on Ms. Baker, but I'm
willing to help if I can.)  See Ya', [-Rick-]

"Rick Lyons" <R.Lyons@_BOGUS_ieee.org> ha scritto nel messaggio news:42a6d305.1760199921@news.sf.sbcglobal.net...
- mostra testo citato -
Thank you Rick for your kind support. Actually I have wrote to Joshua Israelsohn, the editor of EDN on Analog (on 20 May 2005) and to Bonnie Baker (on 30 May 2005) asking them to publish my article but they did not answer!
I wrote also to Jennifer Huber (on 31 May 2005), managing Editor of circuitcellar.com, but the same no answer. Eventually I decided to publish the article on the newsgroups and in a site trying to spread it around.
Here it is my last letter:
Da: Angelo Ricotta
Data: 05/31/05 15:20:54
A: jennife...@circuitcellar.com
Oggetto: Article Proposal

Dear Jennifer

Stimulated by a recent article on undersampling published on EDN, I thought it would be of interest for people working on signal processing area to read about an intriguing story concerning undersampling and that involved me in the past.

The article is in the attachment. Let me know if it is of your interest.

Yours sincerely
Angelo Ricotta

If you think you may help, try. You have my gratefulness for that.
See you
Angelo